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THE EFFECT OF VARIABLE TRANSPORT PROPERTIES ON A DISSOCIATED BOUNDARY LAYER WITH SURFACE REACTION

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Abstract—There is carried out a study of the alteration in surface heat transfer and in the profiles of the flow variables associated with several descriptions of the transport properties of a dissociating gas. A boundary layer of the similar type is considered; a reasonably accurate description of the viscosity, conductivity and diffusivity of a mixture of atoms and molecules is employed along with several approximate descriptions which have been employed in the past. Numerical examples corresponding to high altitude hypersonic flight are considered and indicate that alterations of heat transfer of up to 60 per cent and of the profiles of possible significance can occur depending on the description of the transport properties employed.

NOMENCLATURE

A_i, B_i, D_i, constants in the transport properties for the i-th species;
A_k, B_k, D_k, E_k, constants of integration in the k-th iteration;
c_p, coefficient of specific heat at constant pressure;
C, normalized product of mass-density and viscosity, $\rho\mu/\rho_e\mu_e$;
$\mathscr{D}$, binary diffusion coefficient;
f, stream function, $f' \equiv u/u_e$;
f_o, Blasius stream function;
g, normalized stagnation enthalpy, $h_s/h_{s,e}$;
G, shear function, cf. equation (15);
h, static enthalpy;
j, index, $j \equiv 0$ for two-dimensional, $j \equiv 1$ for axisymmetric;
k_w, specific reaction constant of the body surface;
$\tilde{m}$, external stream parameter, $u_e^2/2h_{s,e}$;
N_o, Avogardro's number;
N_h, heat-transfer parameter, cf. equation (14);
N, number of species;
p, pressure;
q_w, heat transfer at the body surface;
Q, collision cross-section;
r, cylindrical co-ordinate of the body;
R_0, universal gas constant;
$\tilde{s}$, similarity variable, cf. equation (6);
S_c, Schmidt number, $\mu/\rho\mathscr{D}$;
T, temperature;
u, velocity component in x-wise direction;
u, velocity component in x-wise direction;
W, molecular weight;
x, co-ordinate along the body surface;
X, mole fraction;
y, co-ordinate normal to the body surface;
Z, normalized mass fraction of atoms α/α_e;
α, mass fraction of atoms;
δ, parameter cf. equation (8);
Δ, constant in enthalpy-temperature relation, cf. equation (7);

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ζ, surface catalyticity cf. equation (12);
η, similarity variable, cf. equation (5);
θ, normalized temperature, T/T_e;
λ, coefficient of heat conductivity;
μ, viscosity coefficient;
ρ, mass density;
σ, Prandtl number.

Subscripts

A, atoms;
e, conditions external to the boundary layer;
k, iteration index;
M, molecules;
s, stagnation conditions;
s, e, stagnation conditions external to the boundary layer;
w, conditions at the wall or body surface.

1. INTRODUCTION

THE HEAT transfer to extended surfaces of hypersonic vehicles in high altitude flight depends on surface recombination of the dissociated gas. At sufficiently high altitudes and sufficiently close to a sharp leading edge gas phase reactions can be neglected so that heterogeneous reactions become dominant. Thus there has in recent years been considerable interest devoted to boundary-layer flows with surface reactions [1–6]. Apparently in all of these studies the transport properties, which pertain to the dissociated gas mixture and which enter into the description of the flow, have been represented in a simple fashion. In particular all have assumed that the $\rho\mu$ ratio and the Prandtl and Schmidt numbers to be constant. With respect to the main item of technical interest, namely, the heat transfer, this simplicity is stated to be justified on the grounds of insensitivity thereof to the transport properties, it being generally assumed that only a 5 to 10 per cent error in heat-transfer results from the assumed transport descriptions.

Accordingly, it is considered of interest to analyse some flows which involve surface reactions and which are essentially simple in all respects except for their transport properties, and to evaluate the effects thereof on not only the surface heat transfer but the boundary-layer profiles as well. Thus the constant pressure flow of a simple dissociating gas with constant surface catalyticity is treated; nitrogen is explicitly considered in the determination of the transport and thermodynamic properties, which are described with reasonable accuracy. Flow conditions corresponding to the flight of a body with nose bluntness at 250 000 ft with velocities corresponding to 20 and 26 000 ft/s are assumed in the numerical analysis.

2. BASIC EQUATIONS

Consider the laminar boundary layer of a dissociating gas under conditions wherein gas phase reactions may be neglected. Assume that the flow external to the boundary layer is uniform and involves a fixed mass fraction of atoms α_e. Now the heterogeneous surface reaction which causes the non-uniformity in the degree of dissociation is generally considered to be of first order so that at the wall, i.e. at the surface $y = 0$

$$\alpha_y \sim \alpha \tag{1}$$

The proportionality factor in this relation is a function of the specific reaction constant k_w and of the diffusion coefficient at the wall conditions, and is thus realistically to be considered a function of the streamwise co-ordinate x. Thus, accurate treatment of the boundary layer with heterogeneous reactions involves a non-similar boundary layer, i.e. one which cannot be described by ordinary differential equations in terms of a variable $\eta = \eta(x, y)$. However, it is frequently assumed that the main features of the flow are retained if a convenient variation with respect to x of this proportionality factor is assumed; in this case the boundary layer is either strictly similar or of a non-similar type amenable to analysis.

For the purposes of examining the effect of a more accurate description of the transport properties of a dissociating gas on the surface heat transfer and on the flow profiles, it is considered sufficient to employ the simplification associated with similar flows. Accordingly, the describing equations for the distributions of

velocity, stagnation enthalpy, and atom mass fraction are conveniently written as

$$[Cf'']' + ff'' = 0 \tag{2}$$

$$[(C/\sigma)g'] + fg' + 2\tilde{m}\,[(C/\sigma)(1 - \sigma)\,f'f'']' + \{C(S_c^{-1} - \sigma^{-1})a_e\,[(h_A - h_M)/h_{s,e}]Z'\}' = 0 \tag{3}$$

$$[(C/S_c)Z' + fZ' = 0 \tag{4}$$

where

$$(\,)' \equiv \mathrm{d}/\mathrm{d}\eta, \quad g \equiv h_s/h_{s,e}, \quad h_s \equiv (u^2/2) + ah_A + (1 - a)h_M, \quad Z \equiv a/a_e, \quad \tilde{m} \equiv (u_e^2/2h_{s,e});$$

where the independent variable is the well-known Levy–Lees variable [7]

$$\eta \equiv \rho_e u_e r^j (2\tilde{s})^{-\frac{1}{2}} \int_0^y (\rho/\rho_e)\,\mathrm{d}y = \eta(x, y); \tag{5}$$

and where the streamwise transformed variable $\tilde{s} = \tilde{s}(x)$ is for this case

$$\tilde{s} = \rho_e \mu_e u_e \int_0^x r^{2j}\,\mathrm{d}x \tag{6}$$

The quantities h_A and h_M are the enthalpies per unit mass of the atoms and molecules, respectively. Finally, the transport properties of the mixture are contained in $C \equiv \rho\mu/\rho_e\mu_e$, in the Prandtl number $\sigma \equiv \mu c_p/\lambda$, and in the Schmidt number $S_c = \mu/\rho\mathscr{D}$ where $\mathscr{D}$ is the atom–molecule diffusion coefficient.

As will be seen below, the transport properties are functions of composition and temperature, i.e. of a and T, so it is essential for the present purposes to relate the prime variables f, g, and Z to the temperature. This may be done as follows: it is reasonable to assume that the coefficient of specific heat at constant pressure per unit mass for both the atoms and molecules is the same and constant over the temperature range of interest. Then

$$h_A \simeq \Delta_A + c_pT \qquad h_M \simeq \Delta_M + c_pT \tag{7}$$

and the definition of g leads to

$$\theta = [g - \tilde{m}f'^2 - Z\delta - (\Delta_M/h_{s,e})]/(c_pT_e/h_{s,e}) \tag{8}$$

where

$$\theta \equiv T/T_e, \quad \delta \equiv (a_e/h_{s,e})(\Delta_A - \Delta_M).$$

It will be convenient for the specification of the boundary conditions to write equation (8) as

$$g_w = \theta_w(c_pT_e/h_{s,e}) + Z_w\delta + \Delta_M \tag{9}$$

Now the boundary conditions for equations (2)–(4) are: at $\eta = 0$,

$$f = f' = 0 \qquad \theta = \theta_w, \text{ given constant,}^* \qquad Z' - \zeta Z = 0,\ \zeta = \text{given constant} \tag{10}$$

and at $\eta \to \infty$,

$$f' = g = Z = 1 \tag{11}$$

The parameter ζ is the so-called surface catalyticity, which is exactly expressible for first order reactions as

$$\zeta = k_wS_{c,w}(2\tilde{s})^{\frac{1}{2}}/\mu_w u_e r^j \tag{12}$$

It is to be noted from equation (12) that the requirement for $\zeta =$ constant for the flows under consideration involves a special variation of k_w with $\tilde{s}$, e.g. $k_w \sim \tilde{s}^{-\frac{1}{2}}$ if $j = 0$.

Of particular interest in the flows in question is the heat transfer to the surface; it is due to thermal conduction and to diffusion and is expressed as

$$q_w \equiv [\lambda T_y + \rho\mathscr{D}\,(h_A - h_M)\,a_y]_w. \tag{13}$$

In terms of the prime variables and of the approximations employed here, equation (13) yields the following form which is convenient for further comparisons:†

$$N_h \equiv [q_w(2\tilde{s})^{\frac{1}{2}}/\rho_e\mu_e u_e r^j h_{s,e}] = (C_w/\sigma_w)\,\{g_w' - Z_w'\,\delta[1 - (\rho\mathscr{D}\sigma/\mu)_w]\} \tag{14}$$

3. METHOD OF SOLUTION WITH ACCURATE TRANSPORT PROPERTIES

If the transport properties are explicitly represented in terms of θ and Z, then equations (2)–(4) can be put in the form of seven first order,

* The specification of θ_w rather than g_w is preferable for this study and for applications.

† Note that the heat-transfer parameter N_h as defined here permits the heat transfer q_w to be computed from free-stream conditions alone. Thus, alterations in wall conditions, e.g. in g_w, C_w, etc. for a given θ_w but for various descriptions of the transport phenomena do not obscure the effect of such descriptions on the heat transfer.

non-linear, ordinary differential equations, whose solution by standard numerical techniques could be carried out without difficulty, except for the split nature of the boundary conditions. However, it is necessary to carry out the integration from $\eta = 0$ with guesses as to f''_w, g'_w, and Z_w and to improve successively these guesses so that as $\eta \to \infty$, $f' = g = Z = 1$. Thus each solution involves a search in a three-space of f''_w, g'_w, and Z_w.

It seems preferable to employ an iteration method of solution related to that previously employed in simpler problems [8, 9].* Consider first equation (2); let $G \equiv Cf''$ so that

$$G' + (f/C)G = 0 \tag{15}$$

Now consider an iterative method of solution such that

$$G'_k + (f/C)_{k-1} G_k = 0$$

i.e. so that (f/C) is assumed known from the previous iteration. Integration twice with $f'_w = 0$ leads to

$$f'_k = A_k \int_0^\eta C_{k-1}^{-1} \exp\left[-\int_0^{\eta'} (f/C)_{k-1}\, d\eta''\right] d\eta' \tag{16}$$

where A_k is a constant of integration determined so that $f'_k(\infty) = 1$ for each iterate. Thus

$$A_k = \left\{\int_0^\infty C_{k-1}^{-1} \exp\left[-\int_0^{\eta'} (f/C)_{k-1}\, d\eta''\right] d\eta'\right\}^{-1} \tag{17}$$

A final integration gives

$$f_k = \int_0^\eta f'_k\, d\eta' \tag{18}$$

The next iteration cannot, of course, be carried out at this point in the analysis since the quantities required to compute C_k, i.e. θ_k and Z_k, are not available.

* In both references 8 and 9 only the momentum equation with $C \equiv 1$ is considered. The details of the method employed here follow more closely that given in reference 9, although the convergence of the method of reference 8 is proven therein. Proof of convergence of the method employed here has not been attempted. It is also noted that several authors in the past have used procedures for treating the two-point boundary value problem based on formal quadrature; indeed, Dr. W. J. Rae has pointed out in a private communication to the second author that R. F. Probstein [10] has used a procedure similar to that of the present report for a simplified energy equation.

It is convenient to consider next the equation of atom concentration, i.e. equation (4); let $Q \equiv (C/S_c)Z'$ and consider an iterative solution so that†

$$Q'_k + (S_c f/C)_{k-1} Q_k = 0 \tag{19}$$

Integration yields

$$Z_k = B_k \int_0^\eta \{(S_c/C)_{k-1} \exp\left[-\int_0^{\eta'} (S_c f/C)_{k-1}\, d\eta''\right]\} d\eta' + D_k \tag{20}$$

where B_k and D_k are selected so that each iterate satisfies the boundary conditions at $\eta = 0, \infty$. There are obtained

$$B_k = \frac{\zeta}{[(S_c/C)_w]_{k-1} + \zeta \int_0^\infty \{\}\, d\eta'} \tag{21}$$

$$D_k = \frac{[(S_c/C)_w]_{k-1}}{[(S_c/C)_w]_{k-1} + \zeta \int_0^\infty \{\}\, d\eta'} \tag{22}$$

Note that $Z_{w,k} = D_k$ and thus that $g_{w,k}$ may at this point in one cycle of the iteration be considered known from equation (9).

A similar treatment of the energy equation, equation (3), can be carried out; let

$$R \equiv (C/\sigma)g' \tag{23}$$

so the equation (3) becomes, according to the iterative point of view,

$$R'_k + (\sigma f/C)_{k-1} R_k = H'_{k-1} \tag{24}$$

where

$$H \equiv 2\tilde{m}(C/\sigma)(1-\sigma)f'f'' + C(\sigma^{-1} - S_c^{-1})\alpha_e \delta Z'.$$

Integration followed by rearrangement yields R_k in a convenient form;‡ namely

$$\left.\begin{aligned} R_k = H_{k-1} - \exp\left[-\int_0^\eta (\sigma f/C)_{k-1}\, d\eta'\right] \\ \left\{\int_0^\eta H_{k-1} (\sigma f/C)_{k-1} \right. \\ \left. \exp\left[\int_0^\eta (\sigma f/C)_{k-1}\, d\eta''\right] d\eta'\right\} - E_k \end{aligned}\right\} \tag{25}$$

† Note that in equation (19) and in the subsequent steps of one iteration cycle, the most recent iterates can be used; e.g. in equation (19), f_k can be used.

‡ Note that throughout this analysis efforts are made to avoid the necessity of differentiating the transport properties; this always appears to be possible.

A final integration yields

$$\left.\begin{aligned} g_k = g_{w,k} &+ \int_0^{\eta} (\sigma/C)_{k-1} \{H_{k-1} \\ &- \exp\left[-\int_0^{\eta'} (\sigma f/C)_{k-1}\, d\eta''\right] \\ &(H_{w,k-1} + \int_0^{\eta'} H_{k-1} (\sigma f/C)_{k-1} \\ &\exp\left[\int_0^{\eta''} (\sigma f/C)_{k-1}\, d\eta'''\right] d\eta'')\} \, d\eta' \\ &+ E_k \int_0^{\eta} (\sigma/C)_{k-1} \\ &\exp\left[-\int_0^{\eta'} (\sigma f/C)_{k-1}\, d\eta''\right] d\eta' \end{aligned}\right\} \quad (26)$$

The boundary condition at $\eta \to \infty$ determines E_k, noting again that $g_{w,k}$ is known; thus

$$E_k = \frac{(1 - g_{w,k}) - \int_0^{\infty} (\sigma/C)_{k-1} \{\ \} \, d\eta'}{\int_0^{\infty} (\sigma/C)_{k-1} \exp\left[-\int_0^{\eta'} (\sigma f/C)_{k-1}\, d\eta''\right] d\eta'} \quad (27)$$

After a set of iterate functions f_k, Z_k and g_k are computed as indicated above, the iterate function θ_k can be computed from equation (8), new iterates for C, σ, S_c can be computed, and a new cycle begun.

4. SOLUTIONS WITH SIMPLE TRANSPORT PROPERTIES

It will be interest for the purposes of the present study to have available the solution to equations (2)–(4) subject to the boundary conditions of equation (10) with the simplified descriptions of the transport properties usually applied. Thus consider first the approximations $C \equiv 1$, and σ, S_c equal to constants $\bar{\sigma}$, $\bar{S}_c$. Then the solution to equation (2) is the well-known Blasius function [11], denoted here as f_0 and the solutions for Z and g may be obtained directly from equations (9), (20)–(22), (26) and (27) with the iteration indices k and $k - 1$ removed. There are obtained

$$Z = \frac{1 + \zeta \int_0^{\eta} (f_o''/f_{o,w}'')^{\bar{S}_c} \, d\eta'}{1 + \zeta \int_0^{\infty} (f_o''/f_{o,w}'')^{\bar{S}_c} \, d\eta'} \quad (28)$$

$$Z_w = \left[1 + \zeta \int_0^{\infty} (f_o''/f_{o,w}'')^{\bar{S}_c} \, d\eta'\right]^{-1} \quad (29)$$

$$g = g_w + 2\tilde{m}(1 - \bar{\sigma})\{(f_0'^2/2) - \bar{\sigma} \int_0^{\eta} (f_0''/f_{o,w}'')^{\bar{\sigma}} \left[\int_0^{\eta'} f_o f_o' f_o'' (f_o''/f_{o,w}'')^{-\bar{\sigma}} \, d\eta''\right] d\eta'\} +$$
$$E_w \bar{\sigma} \int_0^{\eta} (f_o''/f_{o,w}'')^{\bar{\sigma}} \, d\eta' + [1 - (\bar{\sigma}/\bar{S}_c)] \{(Z - Z_w) - Z_w' \int_0^{\eta} (f_o''/f_{o,w}'')^{\bar{\sigma}} \, d\eta'$$
$$- \bar{\sigma} \int_0^{\eta} (f_o''/f_{o,w}'')^{\bar{\sigma}} \left[\int_0^{\eta'} (f_o''/f_{o,w}'')^{-\bar{\sigma}} f_o Z' \, d\eta''\right] d\eta'\} \quad (30)$$

$$E_w = \frac{(1 - g_w) - 2\tilde{m}(\bar{\sigma} - 1)\{1 - \int_0^{\infty} (f_o''/f_{o,w}'')^{\bar{\sigma}} [\,] \, d\eta'\}}{\bar{\sigma} \int_0^{\infty} (f_o''/f_{o,w}'')^{\bar{\sigma}} \, d\eta'} \quad (31)$$

where equations (9) and (29) give g_w.

An alternate approximation for the transport properties involves $S_c = \bar{S}_c = \sigma = \bar{\sigma} = 1$ but with C variable. In this case equations (3) and (4) and the boundary conditions thereon are satisfied by Crocco-type algebraic relations among f, g and Z. It is easy to show that

$$g = g_w + (1 - g_w) f' \quad (32)$$

$$Z = (f_w'' + \zeta f')/(f_w'' + \zeta) \quad (33)$$

In equations (32) and (33) f' and f_w'' must be obtained from the solution of equation (2) with, in general, C a variable. Note that in this case C can be represented as a function of f' alone by use of equations (8), (32) and (33), and that the iteration method of solution leading to equation (16) can be applied for the solution of equation

(2) alone. Finally, g_w is given by equation (9) with θ_w specified and with Z_w obtained from equation (33).

5. TRANSPORT PROPERTIES FOR NUMERICAL EXAMPLES

To make the comparison between the solutions with accurate and with simple transport properties, it is assumed here that the dissociating gas is nitrogen and that the molecules and atoms behave as rigid spheres with constant collision cross-sections. In addition, the mixture properties have been computed according to the approximate equations which are due to Fay [12] but which are similar to those employed in the past.

From the equation of state for a binary mixture

$$\frac{\rho}{\rho_e} = \frac{1 + \alpha_e}{1 + \alpha_e Z}\theta^{-1},\ \rho_e = (pW_1/R_oT_e)\,(1 + \alpha_e)^{-1} \tag{34}$$

From Fay, in general, for a mixture of N species

$$\mu = \sum_{i=1}^{N} \frac{X_i\,\mu_i}{\sum_{k=i}^{N} X_k\,G_{ik}} \tag{35}$$

where

$$G_{ii} = 1$$

$$G_{ik} = 1{\cdot}385\,\mu_i\,R_o\,T/p\,W_i\,\mathscr{D}_{ik},\ (i \neq k) \tag{36}$$

When equation (35) and (36) are applied to a binary mixture resulting from dissociation, then $i, k = 1, 2$ and

$$\mathscr{D} \equiv \mathscr{D}_{12} = BT^{3/2}\,R_0/p\,W_1$$

$$B = (3/32)\,(12\,R_o\,W_1\,\pi)^{1/2}/N_o\,Q_{12} \tag{37}$$

where Q_{12} is the collision cross-section for atoms and molecules, and N_o is Avogardro's number. For a dissociating gas Q_{12} may be computed according to the combination rule

$$Q_{12} = \pi\,\sigma_{12}^2 = \pi\,(\sigma_1 + \sigma_2)^2/4 \tag{38}$$

where σ_1 and σ_2 are the effective molecular diameters of atoms and molecules, respectively, taken here to be constants. Moreover, in accordance with the aforementioned approximations for the properties of the molecules and atoms,

$$\mu_i \simeq A_i\,T^{\frac{1}{2}}\ i = 1,2 \tag{39}$$

where

$$A_1 = 2{\cdot}7(10^{-5})\pi(W_1)^{\frac{1}{2}}/Q_{11},\ \text{g/cm s}$$

$$A_2 = 2{\cdot}7(10^{-5})\pi(2W_1)^{\frac{1}{2}}/Q_{22},\ \text{g/cm s}$$

provided the cross sections are expressed in the units cm^2. Thus, from equation (35)

$$\mu \simeq T^{\frac{1}{2}}\left\{\frac{A_1}{1 + 1{\cdot}385[A_1(1 - \alpha_e Z)/2\,B\alpha_e Z]} + \frac{A_2}{1 + 1{\cdot}385[A_2\alpha_e Z/B(1 - \alpha_e Z)]}\right\} \tag{40}$$

Equation (40) applied to external conditions, i.e. with $T = T_e$, $Z = 1$, determines μ_e. Thus equations (34) and (40) permit

$$C = C(\theta, Z) = C(f', g, Z)$$

to be obtained.

In a similar manner, after Fay, the conductivity of a mixture can be computed* and thus the Prandtl and Schmidt numbers can be expressed in terms of T [and thus through equations (18) in terms of g, f and Z] and of Z. The following thermodynamic and molecular parameters have been employed in the numerical examples:

$$\left.\begin{aligned} \sigma_1 &= 2{\cdot}2(10^{-8})\ \text{cm} \\ \sigma_2 &= 3{\cdot}0(10^{-8})\ \text{cm} \\ \Delta_A &= 8{\cdot}0(10^3)\ \text{cal/g} \\ \Delta_M &= -\,1{\cdot}8(10^2)\ \text{cal/g} \\ c_p &= 0{\cdot}32\ \text{cal/g degK} \\ W_1 &= 14\ \text{g/mol} \end{aligned}\right\} \tag{41}$$

Where possible the thermodynamic and transport properties of dissociated nitrogen, as predicted by the equations and parameters presented here, have been compared with more accurate calculations; the present analysis has been found to yield satisfactory results except for a mixture which is essentially undissociated,

* An approximate Eucken correction consistent with the assumption of fully excited vibrational energy for the molecular species has been employed.

i.e. $\alpha_e \simeq 0$ and which is not of interest to the present study.

6. FLOW CONDITIONS AND COMPUTING PROCEDURES FOR NUMERICAL EXAMPLES

To examine the influence of the transport properties on the boundary layer under consideration and under conditions of high speed flight of possible interest, the properties external to the boundary layer on a blunted slab are estimated as follows: an undisturbed stream is characterized by a static pressure, taken to be $2(10^{-5})$ atm so that it corresponds to an altitude of roughly 250 000 ft, and by a velocity. Then, employing strong shock approximations and the assumption of equilibrium dissociation behind the bow shock, the state of the dissociated gas at the blunt nose is readily found. It is then assumed that an expansion corresponding to frozen chemistry back to ambient pressure occurs so that α remains constant equal to α_e, and so that u_e (and thus $\tilde{m}$) and T_e can be determined. The resulting values for the external conditions are rounded off to reflect their illustrative nature. It is noted that the radius of the nose is here being considered sufficiently large and the distance down-stream of the nose sufficiently small so that the characteristics of the flow external to the boundary layer are determined by the flow which has passed through the normal shock at the nose. Two flight velocities corresponding to 20 and 26 000 ft/s have been considered.

The iteration solution as described above was programmed for the IBM 650 of the Polytechnic Institute of Brooklyn, employing Simpson's rule for the quadratures with an increment in η of 0·1. The infinity conditions were imposed at $\eta = 6$. The initial distributions for f, C, σ, and S_c were computed from the Crocco-type relations, i.e. from equations (32) and (33) so that the Blasius function was the only input function. Convergence of the iteration was considered satisfactory when the functions f, g, and Z and their significant derivatives differed in two successive iterations by less than 10^{-3} at several η points, including $\eta = 0$, i.e. the wall. It was found that six to eight iterations, taking roughly 20 to 25 min, was required to meet this criterion, which might be considered excessively severe for present purposes.

7. RESULTS AND DISCUSSION

The principal results of the nine cases studied are presented in Table 1.* Listed initially are the values of α_e, $\tilde{m}$, T_e, T_w and ζ characterizing the flow and surface conditions. Listed subsequently are the parameters g_w, g_w', f_w'', Z_w', and N_h characterizing the solutions and the parameters C_w, $S_{c,w}$, σ_w characterizing the transport properties at the wall; this listing is given for the solutions obtained from accurate transport properties and from various approximate descriptions thereof.

Presented in Fig. 1–5 are representative profiles obtained for the cases whose wall values are listed in Table 1. They have been selected to indicate the main effects of the description of the transport properties on the flow profiles. It is noted that the profiles have been presented in terms of the parameters which lead to the Blasius function in the special case $C = \bar{\sigma} = \bar{S}_c = 1$; this presentation tends to reduce the effect of

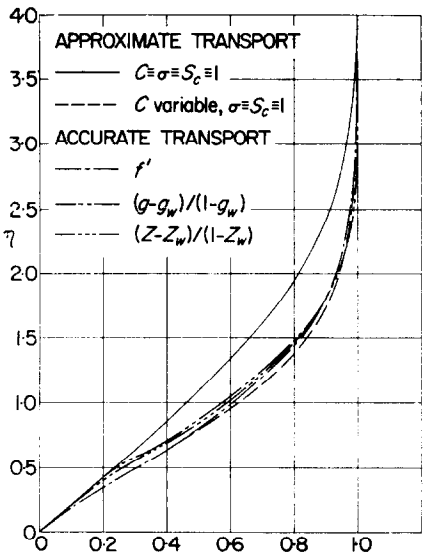

FIG. 1. Profiles for case 2: $\alpha_e = 0{\cdot}75$, $\tilde{m} = 0{\cdot}189$, $T_e = 400°K$, $T_w = 1000°K$, $\zeta = 0{\cdot}2$.

* Note that cases 1–5 evidently pertain to the higher of the two flight velocities considered.

† Note that Z_w can be computed from Z_w' and ζ according to equation (10).

Table 1. Summary of numerical results

	Flow parameters					$C = \sigma = S_c = 1$					C Variable, $\sigma = S_c = 1$					
Case	a_e	$\tilde{m}$	T_e, °K	T_w, °K	ζ	g_w	g_w'	f_w''	Z_w'	N_h	g_w	g_w'	f_w''	Z_w'	N_h	C_w
1	0·75	0·189	400	300	0·2	0·563	0·205	0·470	0·141	0·205	0·473	0·152	0·290	0·118	0·195	1·280
2	0·75	0·189	400	1000	0·2	0·593	0·191	0·470	0·141	0·191	0·607	0·201	0·511	0·144	0·135	0·671
3	0·75	0·189	400	300	1	0·251	0·351	0·470	0·319	0·352	0·148	0·205	0·241	0·194	0·326	1·59
4	0·75	0·189	400	1000	1	0·281	0·337	0·470	0·319	0·338	0·269	0·321	0·440	0·305	0·258	0·804
5	0·75	0·189	400	1000	10	0·056	0·443	0·470	0·449	0·443	0·048	0·348	0·366	0·353	0·347	0·996
6	0·4	0·264	600	300	0·2	0·496	0·237	0·470	0·141	0·237	0·396	0·157	0·259	0·113	0·250	1·59
7	0·4	0·264	600	1000	0·2	0·546	0·213	0·470	0·141	0·213	—	—	—	—	—	—
8	0·4	0·264	600	300	1	0·216	0·368	0·470	0·319	0·368	—	—	—	—	—	—
9	0·4	0·264	600	1000	1	0·267	0·345	0·470	0·319	0·344	—	—	—	—	—	—

	$C = 1$, $\sigma = S_c = 0·7$					Accurate transport							
Case	g_w	g_w'	f_w''	Z_w'	N_h	g_w	g_w'	f_w''	Z_w'	N_h	C_w	$S_{c,w}$	σ_w
1	0·541	0·177	0·470	0·135	0·253	0·450	0·134	0·299	0·113	0·227	1·29	0·806	0·681
2	0·571	0·165	0·470	0·135	0·236	0·595	0·172	0·524	0·141	0·145	0·673	0·887	0·662
3	0·229	0·306	0·470	0·293	0·437	0·116	0·164	0·246	0·155	0·411	1·63	0·629	0·757
4	0·259	0·294	0·470	0·293	0·420	0·227	0·260	0·440	0·254	0·319	0·832	0·664	0·736
5	0·059	0·380	0·470	0·397	0·543	0·041	0·265	0·376	0·265	0·432	1·00	0·589	0·785
6	0·476	0·199	0·470	0·135	0·285	0·368	0·136	0·266	0·105	0·373	1·62	0·675	0·731
7	0·526	0·178	0·470	0·135	0·255	0·520	0·177	0·462	0·133	0·212	0·845	0·705	0·717
8	0·197	0·315	0·470	0·293	0·450	0·096	0·168	0·236	0·156	0·487	1·88	0·606	0·773
9	0·247	0·294	0·470	0·293	0·420	0·213	0·255	0·410	0·250	0·363	0·989	0·621	0·762

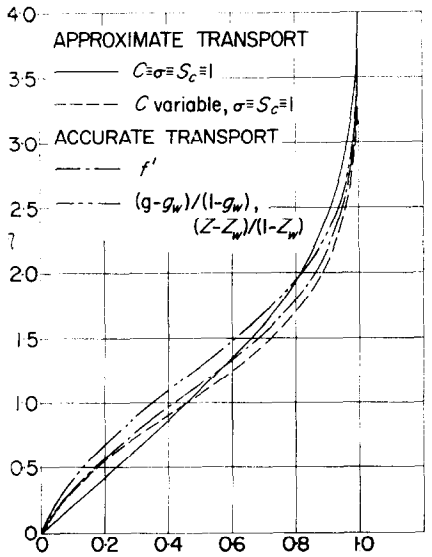

FIG. 2. Profiles for case 3: $a_e = 0{\cdot}75$, $\tilde{m} = 0{\cdot}189$, $T_e = 400°\text{K}$, $T_w = 300°\text{K}$, $\zeta = 1$.

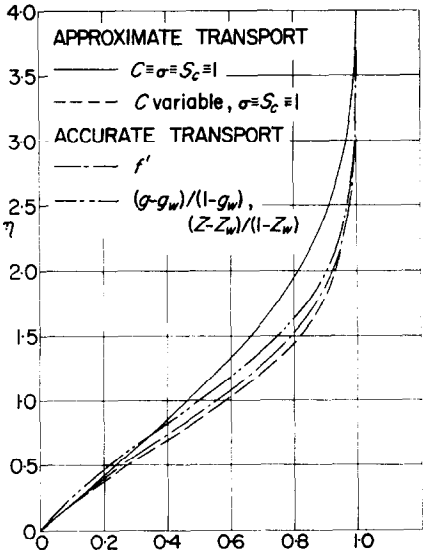

FIG. 3. Profiles for case 4: $a_e = 0{\cdot}75$, $\tilde{m} = 0{\cdot}189$, $T_e = 400°\text{K}$, $T_w = 1000°\text{K}$, $\zeta = 1$.

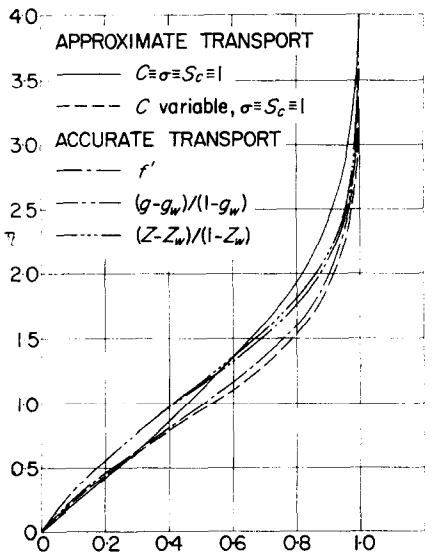

FIG. 4. Profiles for case 5: $a_e = 0{\cdot}75$, $\tilde{m} = 0{\cdot}189$, $T_e = 400°\text{K}$, $T_w = 1000°\text{K}$, $\zeta = 10$.

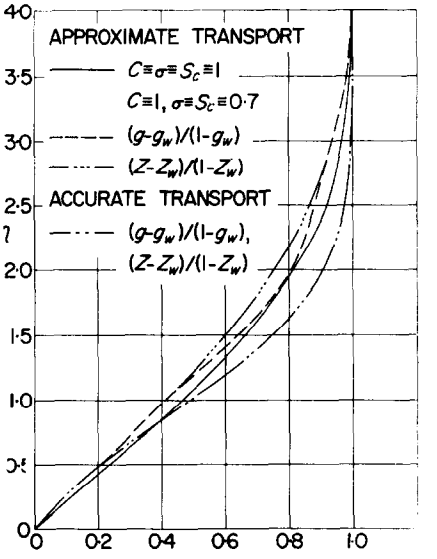

FIG. 5. Profiles for case 4: $a_e = 0{\cdot}75$, $\tilde{m} = 0{\cdot}189$, $T_e = 400°\text{K}$, $T_w = 1000°\text{K}$, $\zeta = 1$.

accurate transport properties in that the effects of altered wall values Z_w and g_w are minimized. Furthermore, it should be appreciated that the presentation of the profiles has been made in terms of the transformed variable η, and that in the physical plane, i.e. in terms of y, greater alterations can occur.

In Figs. 6 and 7 there are compared respectively values of the heat transfer parameter N_h and the wall enthalpy g_w as predicted by the

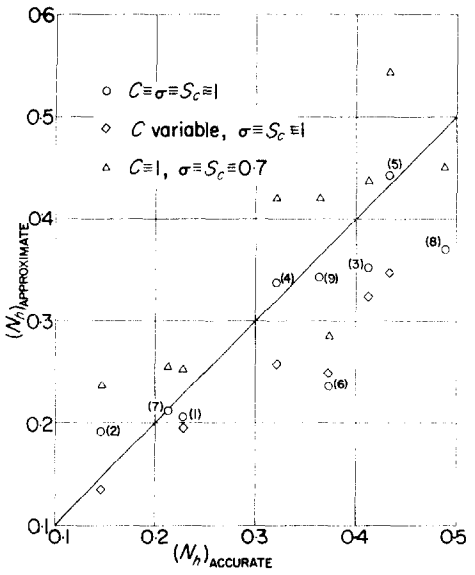

FIG. 6. Comparison of predicted heat-transfer rate.

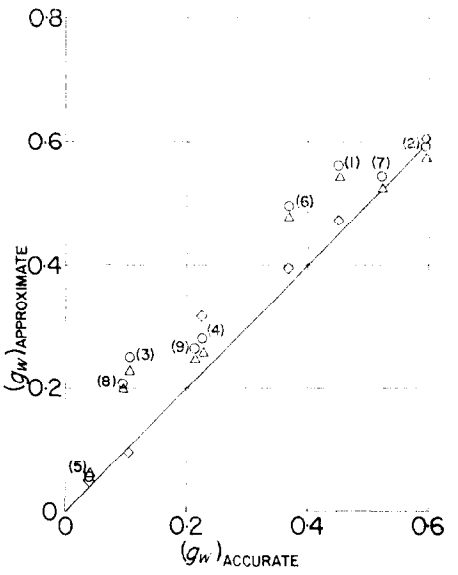

FIG. 7. Comparison of predicted wall enthalpy.

solutions with accurate and approximate transport properties.

Examination of Table 1 and Figs. 6 and 7 indicates that significant alterations in wall values, e.g. in heat transfer and in g_w result from accurate descriptions of the transport properties. The present errors in heat transfer tend to increase with α_e and T_w but are relatively insensitive to ζ; e.g. for case 2 errors in heat transfer, as indicated by N_h, of from 5 to 60 per cent arise because of use of approximate transport properties, whereas for case 8 the maximum error is less than 25 per cent. It would thus appear from these results that the frequently stated relative insensitivity of boundary-layer characteristics to transport properties may not be valid.

Examinations of the profiles of the flow parameters as predicted by the accurate and approximate transport representations, and as shown in Figs. 1–5, indicate that differences do arise therefrom. The significance of these alterations would depend, of course, on the purpose of the analysis. However, in experiments concerned with boundary layers involving surface reactions the present results imply that comparison should be made with analyses based on reasonably accurate descriptions of the transport properties.

It is interesting to note that none of the approximate methods appears to be significantly more accurate than another, although a slight preference seems to prevail for the approximation C variable and $\sigma = S_c = 1$; this is not the case usually treated since the velocity field is not given by the Blasius solution.*

8. CONCLUSIONS

There has been considered the laminar boundary layer of a dissociating gas with surface reaction and with no gas phase reaction. Simplifying assumptions with respect to the chemical properties of the surface have been employed so as to yield a boundary layer of the similar type. Of primary interest is the examination of the effect of the transport properties of the gas on the heat transfer to the surface and on the profiles of the flow quantities within the boundary layer. The results for a reasonably accurate description of the viscosity, conductivity and diffusivity of a dissociating gas applied to

* As a result of a careful examination of an earlier version of this paper, Dr. D. E. Rosner has informed the second author that he is able to provide a simple correction to the heat-transfer coefficient N_h for the effects of σ, $S_c \neq 1$ but not for variable C and thus that ". . . the approximate method which does least violence to the $\rho\mu$ variation will be most accurate . . .".

conditions representing flight at high altitudes are compared to those obtained from frequently employed, approximate descriptions of the transport properties. An iterative method of solution of the non-linear describing equations is presented; experience therewith for the problem under consideration indicates that rapid convergence is achieved.

Comparison of the heat transfer, of the surface properties, and of the profiles as obtained by the accurate and approximate representations of the transport properties indicates that under the assumed conditions and according to the analysis here there can occur changes in the surface heat transfer of up to 60 per cent and in the profiles of the flow variables of some significance.

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Résumé—On a conduit une étude de la modification du transport de chaleur pariétal et des profils des variables de l'écoulement associée à plusieurs descriptions des propriétés de transport d'un gaz dissocié. Une couche limite du type en similitude est considérée; une description raisonnablement précise de la viscosité, de la conductivité et de la diffusivité d'un mélange d'atomes et de molécules est utilisée en même temps que plusieurs descriptions approchées qui ont été employées dans le passé. Des exemples numériques correspondant à un vol hypersonique à grande altitude sont considérés et indiquent que des modifications du transport de chaleur allant jusqu'à 60% et d'importance possible pour les profils peuvent se produire selon la description employée des propriétés de transport.

Zusammenfassung—Diese Arbeit befasst sich mit der Änderung des Wärmeübergangs an Oberflächen und der Profile der Strömungsvariablen in Verbindung mit einigen Beschreibungen der Transporteigenschaften eines dissoziierenden Gases. Es wird eine Grenzschicht von gleicher Art betrachtet. Für die Zähigkeit, die Leit- und Diffundierfähigkeit eines Gemisches aus Atomen und Molekülen werden genügend genaue Angaben verwendet neben mehreren Näherungsangaben, die früher in Gebrauch waren. Numerische Beispiele, die einem Hyperschallflug in grosssen Höhen entsprechen, werden in Betracht gezogen und zeigen, dass unter Umständen bedeutende Änderungen des Wärmeübergangs von bis zu 60% und der Profile auftreten können, die von den Angaben über die verwendeten Transporteigenschaften abhängen.

Аннотация—Проведено исследование изменений поверхностного теплообмена и профилей параметров потока в зависмосити от используемых выражений для коэффициентов переноса диссоциирующего газа. Рассмотрен пограничный слой автомодельного типа.

Используется достаточно точное выражение коэффициентов вязкости, теплопроводности и диффузии для смесей атомов и молекул наряду с приближенными описаниями, применявшимися ранее. Рассмотрены численные примеры, соответствующие условиям гиперзвукового полета на большой высоте. Указываестя, что возможны различные значения теплообмена до 60% и существенные различия профилей в зависимости от используемых выражений коэффициентов переноса.